On a Two-sphere Problem and the Radius of the Muon

A. K. Mitra

Department of Mathematics, Texas Tech. University, Lubbock, Texas 79409

(Z. Naturforsch. **31 a**, 1425–1426 [1976]; received August 15, 1976)

The 2p-1s transition energies for a model of two bound spheres have been calculated. A hint towards the estimation of the muon radius has thus been given.

Before high precision measurements of muonic X-rays were carried out by well-known experimental groups [1,2] in 1975, theoreticians were seriously concerned with the apparent discrepancy between theory and experiment. Consequently some basic aspects of quantum electrodynamics (QED) became doubtful. In spite of substantial theoretical work for obtaining those transition energies accurately, including all QED correction terms, much speculation was offered for the resolution of this discrepancy (see, for example, the excellent review by Watson and Sundaresan [3]). In the meantime, motivated from a different point of view [4], the author suggested another approach, which if at all true, may perhaps be best verified from the muonic X-ray transition, since the deep-lying orbits of the muon in a muonic atom may penetrate well into the nucleus. The idea thereby was to attribute a finite geometric shape also to the orbiting particle, and to study the effect of finite size of the muon on its energy levels (as was done for the nucleus with the muon regarded as a point particle [4]).

Although the 1975 experiment has finally removed the discrepancy, mentioned earlier, the author deems it still interesting to, at least from a theoretical view point, study the above effect by means of a constructive model. It is also hoped that such models can be subjected to critical tests when future experimental techniques yielding even higher precision are available.

The aim of this note is therefore to present, following the methodology described in I, a few specific results from the two-sphere model for 2p-1s transition energies for certain high Z nuclei such as Pb and Hg. These results can not be taken too seriously, since the muon should be more realistically treated by the Dirac equation rather than the Schroedinger equation as assumed in I. A variational method for the approximate calculation of the eigenvalues of the Dirac operator for such an extended model is under consideration and will be detailed in a future publication. Thereafter, the other QED corrections [3] may be added as usual.

As in I, we consider here the uniform model where both the nucleus and the muon are considered as uniformly charged spherical clouds of radii R_N and R_μ, respectively, capable of overlaping without deformation, and interacting via electrostatic forces. Since the muon is regarded as a heavy electron, whose inner orbits penetrate the nucleus, the three following geometric configurational cases are possible: Case I: the muon totally within the nucleus, Case II: they partially overlap and Case III: the muon is totally outside the nucleus. The relevant potential $V(\varrho)$ can be calculated by the conventional method in potential theory and is given by:

$$
\begin{aligned}
V(\varrho) &= V_0(C_0 - \varrho^2) \text{ for Case I } (0 \leqq \varrho \leqq 1-\lambda) \\
&= \frac{3V_0}{8\lambda^3}\left(\frac{C_1}{\varrho} + \sum_{K=0}^{5} C_{K+2}\,\varrho^k\right) \\
&\quad \text{for Case II } (1-\lambda \leqq \varrho \leqq 1+\lambda) \\
&= V_0/\varrho \text{ for Case III } (\varrho \geqq 1+\lambda)\,.
\end{aligned}
$$

Here $\varrho = r/R_N$, r being the radial polar coordinate of the muon center relative to the center of the nucleus as the pole, and the coefficients are given by

$$
\begin{aligned}
&C_0 = (\tfrac{3}{2} - \tfrac{3}{10}\lambda^2)\,; \quad C_1 = (1 - 9\lambda^2 + 16\lambda^3 - 9\lambda^4)/12\,; \\
&C_2 = (-2 + 10\lambda^2 + 10\lambda^3 - 2\lambda^5)/5\,; \\
&C_3 = (3 + 6\lambda^2 + 3\lambda^4)/4\,; \\
&C_4 = (-2 - 9\lambda^2 - 2\lambda^3)/3\,; \\
&C_5 = (1+\lambda^2)/4\,; \quad C_6 = 0\,; \quad C_7 = -1/60\,; \\
&V_0 = -Z e^2/R_N\,; \quad \lambda = R_\mu/R_N\,.
\end{aligned}
$$

With this potential, the computation of the energy levels has been carried out exactly in the same way as given in I, except for the following changes: (i) the integrals needed to compute the matrix elements $G_{nn'}$, in I, should be from 0 to $1+\lambda$, and (ii) the Gauss quadrature has been used to compute these integrals correctly to eight decimal places.

The energy levels depend essentially on two parameters, viz, λ, defined above and $\alpha = Z R_N/a$ where $a = 255.92$ F, the first Bohr radius for the muon, and $R_N = 1.2 \times A^{1/3}$ F. The radius R_μ of the muon has been taken arbitrarily as $R_\mu = 1.2\,k$ F, where k is a parameter which has been chosen as 10^{-2} for the purpose of illustration. The 2p-1s transition

Table I. 2p-1s transition energies ε_{spH} and ε_{PT} for spherical and point muon, respectively (in keV) with the arbitrarily fixed radius $R_\mu = 1.2 \times 10^{-2}$ F.

	ε_{PT}	ε_{spH}
$^{208}_{82}$Pb	6448.24	6445.30
$^{202}_{82}$Pb	6409.40	6406.35
$^{202}_{80}$Hg	6175.65	6172.71

energies ε_{spH} are given below and compared with the corresponding point-muon case ε_{PT}. There is about 3 keV contribution to the above transition energies due to the finiteness of the muon. An acceptable estimate of the muon radius, however, will depend on the solution of the Dirac equation and the discrepancy, if any, between the theoretical results and even more precise future measurements. Finally, it may be added that the Dirac equation for the above case may possibly be solved perturbatively from the existing solution [5] for the point muon case, by considering λ as a perturbation parameter.

The author wishes to thank Professor Benjamin S. Duran for a critical reading of the manuscript.

[1] M. S. Dixit, E. P. Hincks, D. Kesseler, J. S. Wadden, C. K. Hargrove, R. J. McKee, H. Mes, and H. L. Anderson, Phys. Rev. Lett. **35**, 1633 [1975].

[2] L. Tauscher, G. Backenstoss, K. Fransson, H. Koch, A. Nilsson, and De Raedt, Phys. Rev. Let. **35**, 410 [1975].

[3] P. J. S. Watson and M. K. Sundaresan, Can. J. Phys. **52**, 203 F [1974].

[4] A. K. Mitra, Z. Naturforsch. **30 a**, 256 [1975], hereafter referred to as I in the text.

[5] R. J. McKee, Phys. Rev. **180**, 1139 [1969].